

A Test of Hořava Gravity: The Dark Energy

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Abstract

Recently Hořava proposed a renormalizable gravity theory with higher spatial derivatives in four dimensions which reduces to Einstein gravity with a *non-vanishing* cosmological constant in IR but with improved UV behaviors. Here, I consider a non-trivial test of the new gravity theory in FRW universe by considering an IR modification which breaks “softly” the detailed balance condition in the original Hořava model. I separate the dark energy parts from the usual Einstein gravity parts in the Friedman equations and obtain the formula of the equations of state parameter. The IR modified Hořava gravity seems to be consistent with the current observational data but we need some more refined data sets to see whether the theory is really consistent with our universe. From the consistency of our theory, I obtain some constraints on the allowed values of w_0 and w_a in the Chevallier, Polarski, and Linder’s parametrization and this may be tested in the near future, by sharpening the data sets.

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Recently Hořava proposed a renormalizable gravity theory with higher spatial derivatives (up to sixth order) in four dimensions which reduces to Einstein gravity with a *non-vanishing* cosmological constant in IR but with improved UV behaviors by abandoning Einstein's equal-footing treatment of space and time [1, 2]. Since then various aspects have been studied. In particular, in [3], it has been pointed out that the black hole solution in the Hořava model does not recover the usual Schwarzschild-AdS black hole even though the general relativity is recovered in IR at the action level. (For another problem in cosmology, see [4].) For this reason, in [5] an IR modification which allows the flat Minkowski vacuum has been studied by introducing a term proportional to the Ricci scalar of the three-geometry $\mu^4 R^{(3)}$ (for related discussions, see also [6]) and recently the *general* black hole and cosmological solutions have been found [7], which reduce to those of [3] in the absence of the IR modification term and those of [5] for vanishing cosmological constant ($\sim \Lambda_W$). (For other aspects, see [8]).

On the other hand, in [7], the author argued that the dark energy may be explained by the Hořava gravity and obtained the equation of state parameter which seems to be consistent with the observational data, by neglecting the matter contributions¹.

In this paper, I consider an improved analysis of the proposal by comparing with the latest data which does not need to know about matter contributions, separately. This would provide a possible test of Hořava gravity.

To this ends, I start by considering the ADM decomposition of the metric

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt) \quad (1)$$

and the IR-modified Hořava gravity action which reads

$$S_g = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2\nu^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2\nu^2} \epsilon^{ijk} R_{il}^{(3)} \nabla_j R^{(3)\ell}_k \right. \\ \left. - \frac{\kappa^2 \mu^2}{8} R_{ij}^{(3)} R^{(3)ij} + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{4\lambda - 1}{4} (R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2 \right) + \frac{\kappa^2 \mu^2 \omega}{8(3\lambda - 1)} R^{(3)} \right] \quad (2)$$

where

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i) \quad (3)$$

is the extrinsic curvature (the dot $(\dot{})$ denotes the derivative with respect to t),

$$C^{ij} = \epsilon^{ikl} \nabla_k \left(R^{(3)j}_l - \frac{1}{4} R^{(3)} \delta^j_l \right) \quad (4)$$

is the Cotton tensor, $\kappa, \lambda, \nu, \mu, \Lambda_W, \omega$ are constant parameters. The last term, which has been introduced in [2, 4, 5], represents a “soft” violation of the “detailed balance” condition in [2] and this modifies the IR behaviors.

¹ Recently, it has been also proposed by Mukohyama [9] that the dark “matter” as integration constant in Hořava gravity, from the “projectability” condition [2]. But in this paper, I consider the non-projectable case, for the definiteness.

Now, in order to study the cosmological implications of the action (2), I consider a homogeneous and isotropic cosmological solution with the standard FRW form²

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2/R_0^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (5)$$

where $k = +1, 0, -1$ correspond to a closed, flat, and open universe, respectively, and R_0 is the radius of spatial curvature of the universe in the current epoch. Assuming the matter contribution to be of the form of a perfect fluid with the energy density ρ and pressure p , I find that [7]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{6(3\lambda - 1)} \left[\rho \pm \frac{3\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{-k^2}{R_0^4 a^4} + \frac{2k(\Lambda_W - \omega)}{R_0^2 a^2} - \Lambda_W^2 \right) \right], \quad (6)$$

$$\frac{\ddot{a}}{a} = \frac{\kappa^2}{6(3\lambda - 1)} \left[-\frac{1}{2}(\rho + 3p) \pm \frac{3\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{k^2}{R_0^4 a^4} - \Lambda_W^2 \right) \right], \quad (7)$$

where the prime ($'$) denotes the derivative with respect to r . I have considered the analytic continuation $\mu^2 \rightarrow -\mu^2$ for $\Lambda_W > 0$ [3, 7, 13] and the upper (lower) sign denotes the $\Lambda_W < 0$ ($\Lambda_W > 0$) case. Note that the $1/a^4$ term, which is the contribution from the higher-derivative terms in the action (2), exists only for $k \neq 0$ and become dominant for small a , implying that the cosmological solutions of general relativity are recovered at large scales. The first Friedman equation (6) generalizes those of [3] and [5] to the case with an arbitrary cosmological constant and the soft IR modification term in [2, 4, 5]. However, it is interesting to note that there is *no* contribution from the soft IR modification to the second Friedman equation (7) and this is identical to that of [3].

If the Friedman equations (6) and (7) are compared with those our universe, expressed in the usual languages of the Einstein gravity with the “unknown” contributions of “dark energy”³,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}(\rho_{\text{matter}} + \rho_{\text{D.E.}}) - \frac{c^2 k}{R_0^2 a^2}, \quad (8)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}[(\rho_{\text{matter}} + \rho_{\text{D.E.}}) + 3(p_{\text{matter}} + p_{\text{D.E.}})], \quad (9)$$

² In some literatures (see [10, 11], for example), it has been claimed that, when considering perturbations around the background metric, the theory does not have an IR limit close to general relativity due to strongly coupled gravity fluctuations. This would be a very important issue for the consistency of the theory though the detailed discussions about this issue (see [12] for the troubles in [10]) is beyond the scope of the present work. But, in practice, this issue might not be quite relevant to our case since there would be a natural low momentum cut-off $\sim \sqrt{|\Lambda_W|}$ when considering fluctuations around cosmological solutions with a non-vanishing but tiny cosmological constant $\sim \Lambda_W$, as favored by the current observational data. This fact may change in our non-relativistic case since the meaning of the horizons would be also changed from the conventional ones, due to the momentum dependence of the light cones in UV. But in IR, i.e., low momentum, the usual meaning of the horizons would be “emerged” from the recovered Lorentz invariance (with $\lambda = 1$) and so does the notion of the low momentum cut-off. Of course, we need some more rigorous analysis for a more explicit confirmation of this fact.

³ I follow the physical convention of Ryden [14] which disagrees with [1, 2]: $G_{\text{Here}} = G_{\text{Horava}}/c^3$, $\Lambda_{\text{Here}} = \Lambda_{\text{Horava}} c^2$.

the energy density and pressure of the dark energy part can be read as (for a related discussion with matters in the context of the original Hořava gravity, see [15])

$$\rho_{\text{D.E.}} = \pm \frac{3\kappa^2\mu^2}{8(3\lambda-1)} \left(\frac{-k^2}{R_0^4 a^4} - \frac{2k\omega}{R_0^2 a^2} - \Lambda_W^2 \right), \quad (10)$$

$$p_{\text{D.E.}} = \mp \frac{\kappa^2\mu^2}{8(3\lambda-1)} \left(\frac{k^2}{R_0^4 a^4} - \frac{2k\omega}{R_0^2 a^2} - 3\Lambda_W^2 \right), \quad (11)$$

respectively, where I have “defined” the fundamental constants of the speed of light c , the Newton’s constant G , and the cosmological constant Λ as

$$c^2 = \frac{\kappa^4\mu^2|\Lambda_W|}{8(3\lambda-1)^2}, \quad G = \frac{\kappa^2 c^2}{16\pi(3\lambda-1)}, \quad \Lambda = \frac{3}{2}\Lambda_W c^2. \quad (12)$$

Note that c^2 is non-negative always⁴, whereas G can be negative, i.e., *anti-gravity*, for $\lambda < 1/3$, which implies that $\lambda_c = 1/3$ is the lower bound for the consistency with our universe. Moreover, with these definitions, one obtain the IR limit of the action (2) as the sum of the λ -deformed Einstein-Hilbert action $S_{\lambda\text{EH}}$ and the IR limit of the dark energy action with

$$S_{\lambda\text{EH}} = \frac{c^4}{8\pi G(3\lambda-1)} \int dt d^3x \sqrt{g} N \left[\frac{1}{c^2} (K_{ij}K^{ij} - \lambda K^2) + R^{(3)} - \frac{2\Lambda}{c^2} \right], \quad (13)$$

$$S_{\text{D.E.}(\text{IR})} = \frac{3c^6\omega}{32\pi G|\Lambda|} \int dt d^3x \sqrt{g} N R^{(3)}. \quad (14)$$

The deformed action $S_{\lambda\text{EH}}$ agrees with the Einstein-Hilbert action when $\lambda = 1$, but otherwise there are explicit λ -dependances, generally⁵. This is in contrast to the directly measurable equations (8), (9) which define c and G , independently of λ ; actually, the Newton’s constant in (8) agrees with what we measure in laboratory, based on the Newton’s Law of Gravity, $F = -GMm/R^2$ [14] and we can observe only the “renormalized” Newton’s constant G , in contrast to [5].

Once the energy density and pressure of dark energy (10), (11) are identified, one can now compute the equation of state parameter as

$$w_{\text{D.E.}} = \frac{p_{\text{D.E.}}}{\rho_{\text{D.E.}}} = \left(\frac{k^2 - 2k\bar{\omega}a^2 - 3\bar{\Lambda}_W^2 a^4}{3k^2 + 6k\bar{\omega}a^2 + 3\bar{\Lambda}_W^2 a^4} \right), \quad (15)$$

⁴ One might include ω term in the definition of c^2 , rather than including in $\rho_{\text{D.E.}}$, by replacing $|\Lambda_W| \rightarrow |\Lambda_W| \pm \omega$ but then c^2 can be negative when $|\omega| > |\Lambda_W|$. This is what has been considered in [5] to get the non-vanishing speed of light for the flat universe limit $\Lambda_W \rightarrow 0$ with $\omega = \pm 8\mu^2(3\lambda-1)/\kappa^2$, though our current universe seems to have a non-vanishing (positive) Λ_W . But actually, there are infinitely many possible definitions of c^2 , depending on how much the ω term contributes to c^2 . In this paper, however, I do not consider all these possibilities and but consider only the simplest choice which is related to that of the Hořava’s original proposal [2, 15]. But this would be justified by experiments basically.

⁵ In [1, 2] and almost all other later works, $S_{\lambda\text{EH}}$ is given by $S_{\lambda\text{EH}} = \frac{c^4}{16\pi G} \int dt d^3x \sqrt{g} N \left[\frac{1}{c^2} (K_{ij}K^{ij} - \lambda K^2) + R^{(3)} - \frac{2\Lambda}{c^2} \right]$ with $c^2 \equiv \frac{\kappa^2\mu^2|\Lambda_W|}{4(3\lambda-1)}$, $G = \frac{\kappa^2 c^2}{32\pi}$, $\Lambda = \frac{3}{2}\Lambda_W c^2$, in the convention of this paper [14]. This agrees with (13) for $\lambda = 1$ but $\lambda < 1/3$ is excluded from a mathematical consistency of $c^2 > 0$, rather than the physical reason of *no anti-gravity*, i.e., $G > 0$ in the definitions of (12).

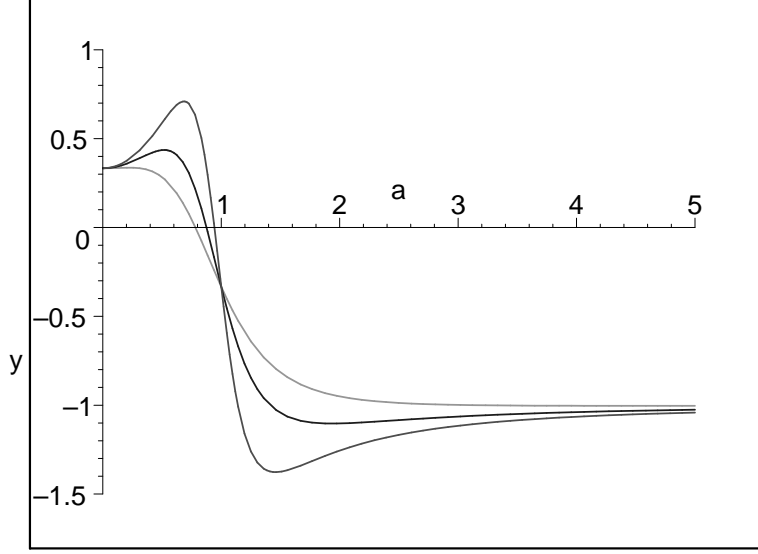


FIG. 1: Plots of equation of state parameters $w_{\text{D.E.}}$ vs. scale factor $a(t)$ for $\bar{\omega}^2 < \bar{\Lambda}_W^2$, $k\bar{\omega} < 0$ ($\bar{\omega} = -1/1.3, -1/2, -1/10$, $k = +1$, or $\bar{\omega} = +1/1.3, +1/2, +1/10$, $k = -1$, with $|\bar{\Lambda}_W| = 1$ (top to bottom in the left region)). When $|\bar{\omega}|$ is not far from $|\bar{\Lambda}_W|$, there is a region where $w_{\text{D.E.}}$ is fluctuating beyond the UV and IR limits. When $|\bar{\omega}|$ is small enough, $w_{\text{D.E.}}$ is monotonically decreasing from $1/3$ in the UV limit to -1 in the IR limit.

where I have introduced $\bar{\omega} \equiv \omega R_0^2$, $\bar{\Lambda}_W = \Lambda_W R_0^2$ for the convenience. This interpolates from $w_{\text{D.E.}} = 1/3$ in the UV limit to $w_{\text{D.E.}} = -1$ in the IR limit but the detailed evolution pattern in between them depends on the parameters $k, \bar{\omega}, \bar{\Lambda}_W$. There are infinite discontinuities when $\bar{\omega}^2 \geq \bar{\Lambda}_W^2$, $k\bar{\omega} < 0$ due to the vanishing $\rho_{\text{D.E.}}$ with a non-vanishing $p_{\text{D.E.}}$ [7]. But physically more interesting case would be $\bar{\omega}^2 < \bar{\Lambda}_W^2$, $k\bar{\omega} < 0$ or $k\bar{\omega} > 0$ where there is no singular point of vanishing $\rho_{\text{D.E.}}$ but smoothly fluctuating/varying between the UV and IR limits (Fig.1,2). For the original Hořava gravity with $\bar{\omega} = 0$, $w_{\text{D.E.}}$ is “always” monotonically decreasing from $1/3$ in the UV limit to -1 in the IR limit [15].

In order to determine $w_{\text{D.E.}}$, I need to know about the constant parameters $k, \bar{\omega}, \bar{\Lambda}_W$. Previously, those have been obtained by neglecting the matter contributions [7]. Here, I consider the more improved analysis which does not need the consideration of the matters separately, based on the latest observational data. To this end, let me consider the series expansion of $w_{\text{D.E.}}$ in (15) near the current epoch ($a = 1$), which coincides with Chevallier, Polarski, and Linder’s parametrization exactly [16], as

$$w_{\text{D.E.}} = w_0 + w_a(1 - a) + w_b(1 - a)^2 + \dots \quad (16)$$

with

$$w_0 = \frac{k^2 - 2k\bar{\omega} - 3\bar{\Lambda}_W^2}{3(k^2 + 2k\bar{\omega} + \bar{\Lambda}_W^2)}, \quad w_a = \frac{8k(\bar{\omega}k^2 + \bar{\omega}\bar{\Lambda}_W^2 + 2k\bar{\Lambda}_W^2)}{3(k^2 + 2k\bar{\omega} + \bar{\Lambda}_W^2)^2}. \quad (17)$$

So, by knowing w_0 and w_a from the observational data, one can determine the constant parameters $\bar{\omega}$ and $\bar{\Lambda}_W$ as

$$\bar{\omega} = \frac{(1 - 2w_0 - 3w_0^2 - w_a)k}{(1 + 4w_0 + 3w_0^2 + w_a)},$$

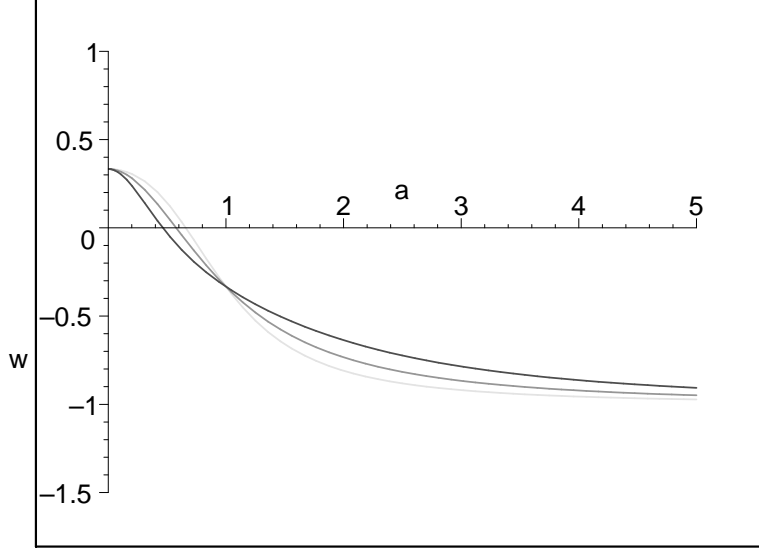


FIG. 2: Plots of equation of state parameters $w_{\text{D.E.}}$ vs. scale factor $a(t)$ for $k\bar{\omega} > 0$ ($\bar{\omega} = \pm 2, \pm 1, \pm 1/2$ (top to bottom in the left region)) for $k = \pm 1$, $|\bar{\Lambda}_W| = 1$. In this case, $w_{\text{D.E.}}$ is “always” monotonically decreasing from $1/3$ in the UV limit to -1 in the IR limit.

$$\bar{\Lambda}_W^2 = \frac{(-1 + 9w_0^2 + 3w_a)k^2}{3(1 + 4w_0 + 3w_0^2 + w_a)}. \quad (18)$$

Note that here I do not need to know about the matter contributions separately, in contrast to the previous analysis [7] in which I have neglected the matter contributions to get the approximate value of $\bar{\Lambda}_W$ from the transition point from deceleration phase to acceleration phase a_T in (7). From the latest data sets (i.e., central values of the best fits) when a non-flat universe is allowed in the analyses [17, 18] $(w_0, w_a) = (-1.10, 0.39), (-1.06, 0.72), (-1.11, 0.475)$, I get $(\bar{\omega}, \bar{\Lambda}_W) = (1.32, 2.44), (1.14, 2.10), (1.30, 2.29)$, respectively and $k = -1$. Once the two constant parameters $\bar{\omega}, \bar{\Lambda}_W$ are determined, the whole function $w_{\text{D.E.}}(a)$ is “completely” determined. From the obtained data, I plot the curves of $w_{\text{D.E.}}(z)$ v.s. the astronomer’s variable of redshift $z = 1/a - 1$ in Fig.3 and these correspond to those of Fig.1 since $|\bar{\omega}| < \bar{\Lambda}_W$. It is interesting to note that these curves appear to give similar results with a nearly model independent analysis of type Ia supernovae (SNe Ia) for $0 \leq z \leq 0.6$ [19] and similar tendencies in other analyses of Gold data sets that use the parametrization (16) even for higher redshifts [20].⁶ In addition, (18) gives some constraints on the allowed values of w_0 and w_a such that $\bar{\Lambda}_W^2 \geq 0$ for the consistency of our theory, $w_a > \frac{1}{3}(1 - 9w_0^2)$, $-1 - 4w_0 - 3w_0^2$ or $w_a < \frac{1}{3}(1 - 9w_0^2)$, $-1 - 4w_0 - 3w_0^2$ (See Fig. 4). This seems to agree with observational data at about 1σ (68.3%) confidence level [17, 20, 21, 23]. This consistency condition may be tested in the near future, by sharpening the data sets.

On the other hand, using the corresponding data $\Omega_k = -0.009, -0.000, -0.0008$ [17, 18] in the current epoch ($a = 1$) for the deviation from the critical density, $\Omega_k \equiv 1 - \Omega_m - \Omega_{\text{D.E.}} =$

⁶ In these analyses, the flat universe has been assumed but the results would be almost the same even for a non-flat universe. This is due to the fact that the relaxation of flatness broadens the ranges of w_0 and w_a but the central values are almost unchanged [17, 18, 21, 22].

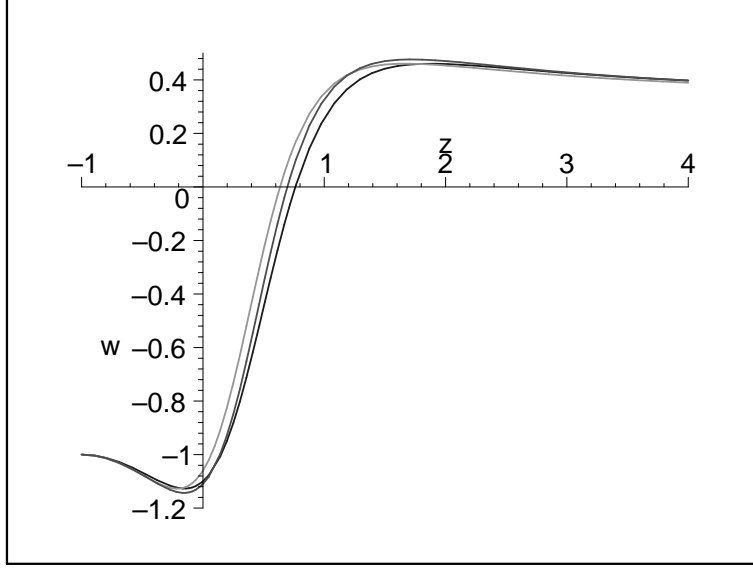


FIG. 3: Plots of equation of state parameters $w_{D.E.}$ vs. redshift $z = 1/a - 1$ for the latest data sets $(\bar{\omega}, \bar{\Lambda}_W) = (1.32, 2.44), (1.14, 2.10), (1.30, 2.29)$ from $(\omega_0, \omega_a) = (-1.10, 0.39), (-1.06, 0.72), (-1.11, 0.475)$ (bottom to top) and $k = -1$.

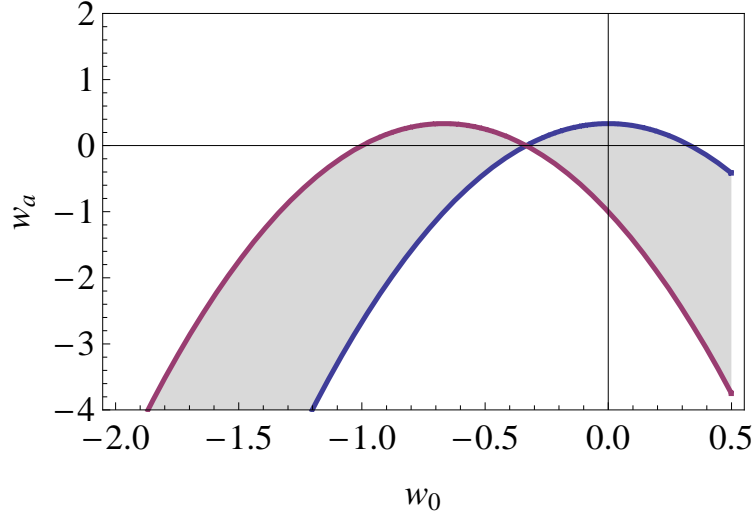


FIG. 4: The ranges of allowed w_0 and w_a (unshaded regions) for the consistency of our theory with (18).

$\mu^2 k |\Lambda_W| L_P^2 / 2a^2 H^2 R_0^2 M_P^2$, Hubble parameter $H \equiv \dot{a}/a$, the ratio of Planck mass and length $M_P/L_P = c^2/8\pi G$, I get $\mu = 0.0013, 0.000, 0.0004$ ($H_0 R_0 M_P/L_P$) with the current value of Hubble parameter H_0 . (See Table 1 for a summary of the data sets and their corresponding constant parameters, in the appropriate units.)

In conclusion, I have considered the dark energy as a possible test of Hořava gravity. It seems that the IR modified Hořava gravity seems to be consistent with the current observational data but we need some more refined data sets to see whether the theory is really consistent with our universe. However, it would be still useful to consider the list of possible

Parameters at $a = 1$	Data analysis Ia [17]	Data analysis Ib [17]	Data analysis II [18]
w_0	-1.10	-1.06	-1.11
w_a	0.39	0.72	0.475
Ω_k	-0.009	-0.000	-0.0008
$\Omega_{\text{D.E.}}$	0.730	0.699	0.739
Ω_{m}	0.279	0.301	0.262
H_0	67.6	65.5	72.4
$\bar{\omega}$	1.32	1.14	1.30
$\bar{\Lambda}_W$	2.44	2.10	2.29
$\bar{\omega}/\bar{\Lambda}_W$	0.542	0.543	0.568
μ	0.0013	0.0000	0.0004

TABLE I: A summary of the data sets *without* assuming the flat universe in a priori and their corresponding constant parameters, in the conventional units of H_0 (km s⁻¹Mpc⁻¹) and μ ($H_0 R_0 M_{\text{P}}/L_{\text{P}}$).

scenarios as follows.

1. If $k = 0$, i.e., flat universe, as predicted by inflationary cosmology but *not* compulsory in the latest analyses [17, 18, 21, 22], is confirmed, there is no effect of the Hořava gravity in the FRW cosmology. But even in this case, its effect to the anisotropic cosmology and non-Gaussianity would be still open problems.

2. If $w_{\text{D.E.}} < -1$ and $k \neq 0$, the original Hořava gravity with the detailed balance condition, which predicts $-1 \leq w_{\text{D.E.}} \leq 1/3$, may be ruled out. According to the current observational data, this scenario seems to be quite plausible and this is also consistent with other theoretical considerations [3, 4, 5, 7].

3. Even if $w_{\text{D.E.}} < -1$, $k \neq 0$, and good agreements for small z are confirmed by determining the constant parameters ω and Λ_W , some disagreements or inconsistencies for higher z can occur; for example, by comparing the higher-order term w_b in (16) with the experiments. In this case, one might consider several further modifications of the Hořava gravity by introducing other detailed-balance breaking terms with the additional constant parameters to control the disagreements. But one does not know how much new terms are needed minimally, in a priori ⁷. Or, one might consider another definition of $\rho_{\text{D.E.}}$ and $p_{\text{D.E.}}$, by considering different definitions of the speed of light, rather than the simplest choice (12), as was discussed in the footnote no.1.

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⁷ For a related discussion, see also [24].

- [1] P. Horava, “Membranes at Quantum Criticality,” JHEP **0903**, 020 (2009) [arXiv:0812.4287 [hep-th]].
- [2] P. Horava, “Quantum Gravity at a Lifshitz Point,” Phys. Rev. D **79**, 084008 (2009) [arXiv:0901.3775 [hep-th]].
- [3] H. Lu, J. Mei and C. N. Pope, “Solutions to Horava Gravity,” arXiv:0904.1595 [hep-th].
- [4] H. Nastase, “On IR solutions in Horava gravity theories,” arXiv:0904.3604 [hep-th].
- [5] A. Kehagias and K. Sfetsos, “The black hole and FRW geometries of non-relativistic gravity,” arXiv:0905.0477 [hep-th].
- [6] T. Sotiriou, M. Visser and S. Weinfurtner, “Phenomenologically viable Lorentz-violating quantum gravity,” arXiv:0904.4464 [hep-th]; “Quantum gravity without Lorentz invariance,” arXiv:0905.2798 [hep-th].
- [7] M. I. Park, “The Black Hole and Cosmological Solutions in IR modified Horava Gravity,” arXiv:0905.4480 [hep-th].
- [8] G. Calcagni, “Cosmology of the Lifshitz universe,” arXiv:0904.0829 [hep-th]; T. Takahashi and J. Soda, “Chiral Primordial Gravitational Waves from a Lifshitz Point,” arXiv:0904.0554 [hep-th]; E. Kiritsis and G. Kofinas, “Horava-Lifshitz Cosmology,” arXiv:0904.1334 [hep-th]; J. Kluson, “Branes at Quantum Criticality,” arXiv:0904.1343 [hep-th]; S. Mukohyama, “Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation,” arXiv:0904.2190 [hep-th]; R. Brandenberger, “Matter Bounce in Horava-Lifshitz Cosmology,” arXiv:0904.2835 [hep-th]; R. G. Cai, L. M. Cao and N. Ohta, “Topological Black Holes in Horava-Lifshitz Gravity,” arXiv:0904.3670 [hep-th]; R. G. Cai, Y. Liu and Y. W. Sun, “On the $z=4$ Horava-Lifshitz Gravity,” arXiv:0904.4104 [hep-th]; Y. S. Piao, “Primordial Perturbation in Horava-Lifshitz Cosmology,” arXiv:0904.4117 [hep-th]; X. Gao, “Cosmological Perturbations and Non-Gaussianities in Horava-Lifshitz Gravity,” arXiv:0904.4187 [hep-th]; E. O. Colgain and H. Yavartanoo, “Dyonic solution of Horava-Lifshitz Gravity,” arXiv:0904.4357 [hep-th]; D. Orlando and S. Reffert, “On the Renormalizability of Horava-Lifshitz-type Gravities,” arXiv:0905.0301 [hep-th]; S. Mukohyama, K. Nakayama, F. Takahashi and S. Yokoyama, “Phenomenological Aspects of Horava-Lifshitz Cosmology,” arXiv:0905.0055 [hep-th]; Y. S. Myung and Y. W. Kim, “Thermodynamics of Horava-Lifshitz black holes,” arXiv:0905.0179 [hep-th]; R. G. Cai, L. M. Cao and N. Ohta, “Thermodynamics of Black Holes in Horava-Lifshitz Gravity,” arXiv:0905.0751 [hep-th]; Y. S. Myung, “Thermodynamics of black holes in the deformed Horava-Lifshitz gravity,” arXiv:0905.0957 [hep-th]; R. G. Cai, B. Hu and H. B. Zhang, “Dynamical Scalar Degree of Freedom in Horava-Lifshitz Gravity,” arXiv:0905.0255 [hep-th]; T. Nishioka, “Horava-Lifshitz Holography,” arXiv:0905.0473 [hep-th]; A. Ghodsi, “Toroidal solutions in Horava Gravity,” arXiv:0905.0836 [hep-th]; R. A. Konoplya, “Towards constraining of the Horava-Lifshitz gravities,” arXiv:0905.1523 [hep-th]; S. Chen and J. Jing, “Strong field gravitational lensing in the deformed Horava-Lifshitz black hole,” arXiv:0905.2055 [gr-qc]; B. Chen, S. Pi and J. Z. Tang, “Scale Invariant Power Spectrum in Horava-Lifshitz Cosmology without Matter,” arXiv:0905.2300 [hep-th]; M. Li and Y. Pang, “A Trouble with Horava-Lifshitz Gravity,” arXiv:0905.2751 [hep-th]; J. Chen and Y. Wang, “Time-like Geodesic Motion in Horava-Lifshitz Spacetime,” arXiv:0905.2786 [gr-qc]; Y. W. Kim, H. W. Lee and Y. S. Myung, “Nonpropagation of scalar in the deformed Horava-Lifshitz

- gravity,” arXiv:0905.3423 [hep-th]; G. Calcagni, “Detailed balance in Horava-Lifshitz gravity,” arXiv:0905.3740 [hep-th]; A. Wang and Y. Wu, “Thermodynamics and classification of cosmological models in the Horava-Lifshitz theory of gravity”, arXiv:0905.4117 [hep-th]; M. Minamitsuji, “Classification of cosmology with arbitrary matter in the Hořava-Lifshitz theory,” arXiv:0905.3892 [astro-ph.CO]; S. Nojiri and S. D. Odintsov, “Covariant Horava-like renormalizable gravity and its FRW cosmology,” arXiv:0905.4213 [hep-th]; M. Botta-Cantcheff, N. Grandi and M. Sturla, “Wormhole solutions to Horava gravity,” arXiv:0906.0582 [hep-th]; Y. S. Myung, “Propagations of massive graviton in the deformed Hořava-Lifshitz gravity,” arXiv:0906.0848 [hep-th]; C. Germani, A. Kehagias and K. Sfetsos, “Relativistic Quantum Gravity at a Lifshitz Point,” arXiv:0906.1201 [hep-th]; A. Ghodsi and E. Hatefi, “Extremal rotating solutions in Horava Gravity,” arXiv:0906.1237 [hep-th]; F. W. Shu and Y. S. Wu, “Stochastic Quantization of the Hořava Gravity,” arXiv:0906.1645 [hep-th]; Y. F. Cai and E. N. Saridakis, “Non-singular cosmology in a model of non-relativistic gravity,” arXiv:0906.1789 [hep-th].
- [9] S. Mukohyama, “Dark matter as integration constant in Horava-Lifshitz gravity,” arXiv:0905.3563 [hep-th].
 - [10] C. Charmousis, G. Niz, A. Padilla and P. M. Saffin, “Strong coupling in Horava gravity,” arXiv:0905.2579 [hep-th].
 - [11] D. Blas, O. Pujolas and S. Sibiryakov, “On the Extra Mode and Inconsistency of Horava Gravity,” arXiv:0906.3046 [hep-th].
 - [12] X. Gao, Y. Wang, R. Brandenberger and A. Riotto, “Cosmological Perturbations in Hořava-Lifshitz Gravity,” arXiv:0905.3821 [hep-th].
 - [13] M. I. Park, “Holography in Three-dimensional Kerr-de Sitter Space with a Gravitational Chern-Simons Term,” *Class. Quant. Grav.* **25**, 135003 (2008). [arXiv:0705.4381 [hep-th]].
 - [14] B. Ryden, “Introduction to cosmology,” *San Francisco, USA: Addison-Wesley (2003)*.
 - [15] E. N. Saridakis, “Horava-Lifshitz Dark Energy,” arXiv:0905.3532 [hep-th].
 - [16] M. Chevallier and D. Polarski, “Accelerating universes with scaling dark matter,” *Int. J. Mod. Phys. D* **10**, 213 (2001) [arXiv:gr-qc/0009008]; E. V. Linder, “Exploring the expansion history of the universe,” *Phys. Rev. Lett.* **90**, 091301 (2003) [arXiv:astro-ph/0208512].
 - [17] K. Ichikawa and T. Takahashi, “The Hubble Constant and Dark Energy from Cosmological Distance Measures,” *JCAP* **0804**, 027 (2008) [arXiv:0710.3995 [astro-ph]].
 - [18] J. Q. Xia, H. Li, G. B. Zhao and X. Zhang, “Determining Cosmological Parameters with Latest Observational Data,” *Phys. Rev. D* **78**, 083524 (2008) [arXiv:0807.3878 [astro-ph]].
 - [19] D. Huterer and A. Cooray, “Uncorrelated Estimates of Dark Energy Evolution,” *Phys. Rev. D* **71**, 023506 (2005) [arXiv:astro-ph/0404062].
 - [20] U. Alam, V. Sahni and A. A. Starobinsky, “The case for dynamical dark energy revisited,” *JCAP* **0406**, 008 (2004) [arXiv:astro-ph/0403687]; U. Alam, V. Sahni and A. A. Starobinsky, “Exploring the Properties of Dark Energy Using Type Ia Supernovae and Other Datasets,” *JCAP* **0702**, 011 (2007) [arXiv:astro-ph/0612381]; A. Shafieloo, “Model Independent Reconstruction of the Expansion History of the Universe and the Properties of Dark Energy,” *Mon. Not. Roy. Astron. Soc.* **380**, 1573 (2007) [arXiv:astro-ph/0703034].
 - [21] Y. Wang and P. Mukherjee, “Observational Constraints on Dark Energy and Cosmic Curvature,” *Phys. Rev. D* **76**, 103533 (2007) [arXiv:astro-ph/0703780].
 - [22] Y. Wang, “Figure of Merit for Dark Energy Constraints from Current Observational Data,” *Phys. Rev. D* **77**, 123525 (2008) [arXiv:0803.4295 [astro-ph]].
 - [23] A. Upadhye, M. Ishak and P. J. Steinhardt, “Dynamical dark energy: Current constraints

- and forecasts,” Phys. Rev. D **72**, 063501 (2005) [arXiv:astro-ph/0411803].
- [24] E. V. Linder and D. Huterer, “How many dark energy parameters?,” Phys. Rev. D **72**, 043509 (2005) [arXiv:astro-ph/0505330].